Developmental Psychology
2016, Vol. 52, No. 6, 867–878

0012-1649/16/$12.00 http://dx.doi.org/10.1037/dev0000113

Aiming to Complete the Matrix: Eye-Movement Analysis of Processing Strategies in Children's Relational Thinking

Zhe Chen
University of California, Davis

Ryan Honomichl
Hiram College

Diane Kennedy
University of California, Davis

Enda Tan
University of British Columbia

The present study examines 5- to 8-year-old children's relation reasoning in solving matrix completion tasks. This study incorporates a componential analysis, an eye-tracking method, and a microgenetic approach, which together allow an investigation of the cognitive processing strategies involved in the development and learning of children's relational thinking. Developmental differences in problem-solving performance were largely due to deficiencies in engaging the processing strategies that are hypothesized to facilitate problem-solving performance. Feedback designed to highlight the relations between objects within the matrix improved 5- and 6-year-olds' problem-solving performance, as well as their use of appropriate processing strategies. Furthermore, children who engaged the processing strategies early on in the task were more likely to solve subsequent problems in later phases. These findings suggest that encoding relations, integrating rules, completing the model, and generalizing strategies across tasks are critical processing components that underlie relational thinking.

Keywords: relational reasoning, encoding relations, integrating rules, completing the model, generalizing strategies

The present study examines the cognitive processing strategies involved in the development and learning of children's matrix problem solving by incorporating a componential analysis, an eye tracking method, and a microgenetic approach. The matrix completion task consists of a grid of objects, with the items placed in the rows varying along one relation (e.g., size) and the items placed in the columns varying along another relation (e.g., shade). The item that should occupy the right corner of the matrix is missing, and the solver's task is to select from a set of alternatives the object that best fits in the empty square in terms of both relations (see Figure 1).

The matrix completion task is essentially a relational reasoning task, and successes and failures on this task parallel the results of other relational reasoning tasks. Relational reasoning refers to the capacity to manipulate in our minds abstract mental representations of relations among objects, attributes, and events (Gentner, 1983; Hummel & Holyoak, 1997). Relational reasoning is central to human thinking and is evident in early childhood as a building block in many areas of higher-order cognition, such as metaphor (Gentner, 1988), analogical problem solving (Chen & Siegler, 2000), spatial reasoning (Namy, Smith, & Gershkoff-Stowe, 1997; Newcombe & Huttenlocher, 2006; Plumert, Ewert, & Spear, 1995; Uttal, 2000), pictorial mapping (Gordon & Moser, 2007; Honomichl & Chen, 2006; Markman & Gentner, 1993; Richland, Morrison, & Holyoak, 2006), symbolic understanding (DeLoache, 1987; Loewenstein & Gentner, 2001; Marzolf & DeLoache, 1994), and scientific reasoning (Chen & Klahr, 1999).

A commonly studied paradigm that requires relational reasoning is the matrix completion task, which has been a central task in both psychometric (Jensen, 1987; Snow, Kyllonen, & Marshalek, 1984) and information-processing theories (Halford, 1993; Carpenter, Just, & Shell, 1990; Sternberg, 1977). Adults' performance on the Raven's Progressive Matrices Test (Raven, Raven, & Court, 1998), a form of the matrix completion task, was highly correlated with other types of relational reasoning tasks, such as numerical and geometric analogies (Jensen, 1987; Snow, Kyllonen, & Marshalek, 1984). Similarly, children who performed well in completing this task tended to do well in solving other tasks involving relational reasoning, such as conservation (Carlson & Wiedl, 1977; Dimitrovsky & Almy, 1975) and seriation (Hamel & van der Veer, 1972).

The examination of children's problem solving on matrix completion tasks can be traced back to Inhelder and Piaget (1964), who found that 8-year-olds outperformed 5-year-olds in successfully solving the problems, while 6- and 7-year-olds' performance was somewhere in between. However, within this latter group, 6-year-olds actually performed somewhat better than 7-year-olds. Subse-

This article was published Online First May 5, 2016.

Zhe Chen, Department of Human Ecology, University of California, Davis; Ryan Honomichl, Department of Psychology, Hiram College; Diane Kennedy, Department of Human Ecology, University of California, Davis; Enda Tan, Department of Psychology, University of British Columbia.

Correspondence concerning this article should be addressed to Zhe Chen, Department of Human Ecology, University of California, Davis, One Shields Avenue, Davis, CA 95616. E-mail: zhechen@ucdavis.edu

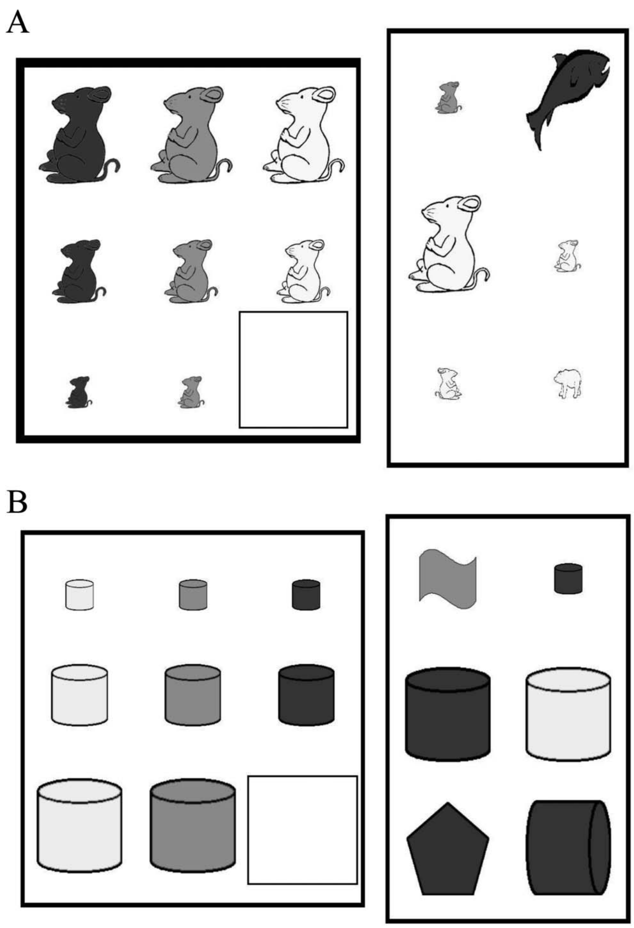

Figure 1. Examples (A and B) of the matrix completion tasks used in the present study.

quent studies have replicated the basic findings of these developmental differences (e.g., Overton & Brodzinsky, 1972; Parker & Day, 1971). This nonlinear path of change may reflect the different strategies that children use at different ages to solve the problem.

While early matrix completion studies tended to focus on overall age differences in performance, more recently Siegler and Svetina (2002) explored the learning processes involved in completing the task. They tested children using a combination of a traditional cross-sectional design (comparing performance in 6- to 8-year-olds) and a microgenetic approach (examining 6-year-olds' learning as they solved matrix completion tasks over several sessions). Their results indicated both improvements in children's performance patterns over the learning sessions as 6-year-olds acquired matrix completion proficiency as well as developmental changes that occurred as the children aged from 6 to 7 years.

Despite the significant contributions of previous findings to our emerging understanding of children's relational reasoning and their performance on matrix completion tasks, we still know little about the specific cognitive components involved as children solve matrix problems and little about the processes by which they learn relational reasoning strategies with experience over short periods of time. In the present studies, we examined the underlying learning processes involved in the matrix completion task for children between the ages of five and eight.

To examine the underlying cognitive processes involved in the task, we conducted a componential analysis of matrix completion performance (Sternberg, 1977). Four key information-processing components are involved in learning to solve matrix problems as a type of relational reasoning task: a) the solver must encode each relevant rule or relation; b) these rules and relations must be integrated so a representation or mental model of the rules can be constructed; c) once the relations are represented, the solver needs to complete the model by searching for the correct item from among the alternatives; and d) the strategies learned in the previous two steps must be generalized to novel isomorphic problems (e.g., Alibali, 1999; Chen & Klahr, 1999; Rittle-Johnson, 2006; Siegler & Chen, 1998, 2008).

Each process presents challenges for learning to solve relational reasoning problems. The *encoding relations* component is a critical initial step toward solving a variety of relational reasoning tasks, and the relationship between successful encoding and subsequent problem solving has been demonstrated in classic information-processing studies (Siegler, 1976; Sternberg, 1977). For example, to solve the balance scale problem (e.g., Inhelder & Piaget, 1958), children need to represent the number of weights placed on each side of the scale as well as the distance of the weights from the fulcrum of the scale, and they need to use both dimensions of information to make accurate predictions about how the balance will move.

The second component involves *integrating rules*. The encoding of separate relations/dimensions needs to be integrated, as evident in the tasks mentioned above as well as in Piagetian conservation and transitive inference tasks (Inhelder & Piaget, 1958, 1964). To solve spatial and symbolic understanding tasks, children first need to represent the spatial relations among the items so as to be able to use this spatial/relational information to guide their later search in the referent space or to integrate items within the same relative position (Loewenstein & Gentner, 2001). Similarly, to solve the matrix completion problem, children must integrate the relations in order to determine which item best completes the matrix.

The third component involves *completing the model* based on the multiple relations represented. The completing the model process operates in different ways depending on the type of relational reasoning problem being solved. For analogical problem solving, for example, the completing the model process involves mapping or aligning the relations between entities, spaces, and structures (e.g., Gentner & Markman, 1997). Searching and aligning problem spaces is a critical process identified by Newell and Simon (1972). According to Gentner's (1983, 1989; Gentner & Markman, 1997) structure-mapping theory, mapping is the process of establishing an alignment of reasonably defined representational structures, such as the solar system and the Rutherford atom (e.g., Gentner, 1983). This process is evident in relational reasoning tasks, such as metaphor, analogical problem solving, transitive inference, spatial reasoning, and symbolic understanding, where a structural alignment between two first-order relations or represented situations is established (DeLoache, 2000; Gentner, 1989; Gentner & Medina, 1998; Loewenstein & Gentner, 2001; Newcombe et al., 1998; Newcombe & Huttenlocher, 2000). The matrix completion task (e.g., Siegler & Svetina, 2002; Sternberg, 1977), as well as the classic analogy (A:B::C:?) (e.g., Goswami & Brown, 1990), is a relational reasoning task with its own specific challenges, which result both from the grid format, with its numerous exemplars along two dimensions, and from the number of alternative choices for completing the matrix. To solve such a problem, participants need to hold the relational representations (e.g., the A:B relation or the matrix rules) in working memory while searching for a match from a response set to complete the C:D relation or the matrix structure. During this completing the model process, solvers need to go back and forth between the Matrix Problem Space and the Response Space as they attempt to eliminate incorrect items and identify and confirm a correct choice.

The fourth component, *generalizing*, is the application of the learned strategies to novel problems. Successful generalization may require applying learned strategies to tasks with different perceptual features, in different forms, and in different contexts (Chen, 2007; Chen & Klahr, 1999; Marzolf & DeLoache, 1994; Rittle-Johnson, 2006; Siegler & Chen, 1998; Siegler & Svetina, 2002). The breadth of generalization can be relatively narrow, especially during the earlier stages of learning and in young children (Chen & Klahr, 2008; Klahr & Chen, 2011).

The first three components, encoding, integrating, and completing, are essential for solving the matrix completion problem as a relational reasoning task, while the fourth process, generalizing, is critical for transferring the processing strategies across isomorphic tasks. Assessing these components and examining the role of each process in problem solving performance should greatly increase our understanding of developmental and individual differences in children's relational thinking and learning. Few studies have directly assessed or measured encoding and mapping in children while they are actively engaged in solving problems. This study accomplishes this assessment and examination by using corneal-reflection eye tracking to pinpoint the components involved in the solution of the matrix completion task. Though this technology has been used for analysis of relational reasoning in adults (e.g., Bethell-Fox, Lohman, & Snow, 1984), it has rarely been used to examine cognitive processing strategies in children's higher-order cognitive tasks.

Beyond the first aim of exploring the processing strategies involved in children's relational reasoning, the second aim of the current research is to examine how young children learn from their problem-solving experience and from feedback to solve relational reasoning problems. Toward this end, the current research is designed with a microgenetic approach, in combination with the eye-tracking approach, to address how young children change their processing strategies after receiving feedback. With traditional behavioral observations of children's verbal responses, it is challenging to examine the underlying processing strategies that children engaged in solving relational reasoning tasks. The use of an eye-tracking approach enables us to assess these online processing strategies and uncover the exact timelines of these cognitive processes. The microgenetic approach involves extensive observation and intensive analyses of processing strategy change as it occurs, with the result that detailed information about the learning process is obtained (Kuhn, 1995; Siegler, 2006; Siegler & Crowley, 1991). Microgenetic studies have shed light on children's strategy change in various domains such as memory strategies, mathematical reasoning, and scientific thinking (Alibali, 1999; Siegler, 2006; Siegler & Jenkins, 1989; Siegler & Svetina, 2002).

In sum, the current research is designed to examine the cognitive components involved in the development and learning of children's relational reasoning. The current study includes several novel features: (a) a new design to assess online problem-solving processing strategies and performance; (b) an eye-tracking approach to assess children's matrix problem solving as they encode relations, integrate rules, complete the model, and generalize strategies across trials/tasks; and (c) a microgenetic approach to examine short-term change in children's strategies and performance. In Experiment 1, we examine developmental differences between a younger group (5- and 6-year-olds) and an older group (7- and 8-year-olds) in matrix problem-solving performance and in the underlying processes involved in solving the tasks. In Experiment 2, we provide at the end of the trial feedback highlighting the relations of the matrix problems, and we examine whether and how the feedback improves young children's performance and their use of processing strategies on subsequent trials.

Experiment 1

Method

Participants. Sixty-nine children with a mean age of 83 months (*SD* = 13.8) participated. Age differences were analyzed by group; 5- and 6-year-olds were categorized as the younger group (ranging from 58 to 84 months, *M* = 71 months *SD* = 7.44) and 7- and 8-year-olds constituted the older group (ranging from 85 to 106 months, *M* = 95 months, *SD* = 5.64). Thirty children (43%) were female. All parents reported normal vision and none of the children wore glasses. An additional six children participated but were dropped from the final sample due to their inability to engage or experimenter's error.

Materials. Children solved 24 matrix completion problems (see Figure 1). Each matrix consisted of a 3-by-3 grid of objects, with the bottom right space left blank. Each matrix was constructed based on two rules, one depicted in the columns, the other in the rows. The rules used to construct matrix problems were *size*, *shape*, *shade*, and *orientation*. In the problem in Figure 1, the Size rule is depicted in the columns and the Shade rule in the rows. To the right of the Matrix Space, a set of six answer options was presented in a separate box—the Response Space. These alternative items were presented so that there was one correct choice (i.e., correct in both rules) and five incorrect items. These incorrect items included three categories of errors: (a) two options that were correct in one relevant rule of the problem (e.g., shade or size) but incorrect in the other relevant rule (e.g., size or shade); (b) one option that was correct in one relevant rule (e.g., shade or size) but incorrect in a rule that was irrelevant to that particular matrix (e.g., orientation); and (c) two items that were incorrect and irrelevant to the matrix completion rules. Half of the matrix puzzles were constructed using concrete objects (e.g., animals, vehicles, tools); the remaining 12 were constructed using abstract shapes. These 24 tasks were presented in two counterbalanced order.

Procedure. Before completing the 24 test problems, children were given a warm-up task consisting of four problems to introduce them to the basic task paradigm. These problems did not require the child to induce any rule or relation as all the objects in the matrix were identical. After the children successfully completed the warm-up, they were seated in front of a 17-inch monitor at a distance of approximately 60 cm. Eye movements were recorded by a Tobii 1750 eye tracker, and their eye movements were calibrated using a five-point procedure.

Following the warm-up problems and calibration, children were then sequentially presented the 24 matrix completion problems. On each trial, the child was asked to take a look at the Matrix Space and come up with an answer to complete the matrix. The experimenter guided the child to attend to the Matrix Space and said to the child: “There are several pictures here (pointing to the objects in both the horizontal and vertical directions), and one is missing here (pointing to the empty box). Which one of these pictures (pointing to the Response Space) should go here (pointing back to the empty box)? Now take a look, and tell me later.” The experimenter then instructed the child to point to his or her choice on the screen using a 10-inch wooden dowel in order to minimize head and body movements. An experimenter seated next to the child keyed the selected answer.

Results

Data reduction and analysis. Fixation analysis and total viewing time served as the basis for examining eye-tracking patterns. Fixations were defined as lasting a minimum of 100 ms. On some trials, little or no eye-tracking data were recorded due to excessive movement. Therefore, on trials in which total gaze duration on both the Matrix Space and the Response Space was under 1,000 ms, all eye-tracking data for that trial was eliminated. However, problem performance was retained. This resulted in the loss of eye-tracking data on 85 trials out of 1,656 (5.1%).

Measures. Areas of interest (AOIs) were created by outlining the entire Matrix Space and the entire Response Space, as well an AOI for each of the eight objects within the Matrix Space and for each of the six options within the Response Space. *Encoding relations* was defined as a series of three fixations that occurred in sequence in the horizontal or vertical direction on objects with the matrix space. For example, if a child looked across an entire matrix row backward or forward or up and down an entire column, this was scored as an encoding event. This variable was calculated as a measure of induction of one of the rules in the problem. A separate measure was *integration of rules*, which involved encoding both dimensions of relations. Integration was measured by two such encoding events within the same trial, with at least one in the vertical direction and one in the horizontal. A third measure involved *completing the model* by searching for a match to the constructed mental model. Following previous studies of eye tracking during visuospatial problems solving (e.g., Bethell-Fox et al., 1984; Gordon & Moser, 2007), we also calculated *Toggling*, which reflected a shift in gaze from the Matrix Space to the Response Space, or vice versa, as a measure of the process related to the completing the model component.

An additional measure included the number of fixations on each item within the Response Space. These options were categorized into four categories: (a) Correct: correct in both rules (the middle right in the Response Space of Example A in Figure 1); (b) Incorrect–One Relevant Rule: incorrect in one relevant rule as compared to the correct item (two such items, top and middle, left); (c) Incorrect–One Irrelevant Rule: incorrect in an irrelevant rule (bottom left); and 4) Incorrect–Relevant and Irrelevant Rules: incorrect in all relevant and irrelevant rules (two such items, top and bottom, right).

Data analyses were performed to address the following central issues: (a) age differences in solving the matrix problems, encoding the relations, integrating the rules, and toggling between the Matrix and Response Spaces while attempting to solve the tasks; and (b) the relations between problem-solving performance and online processing strategies at each age level.

Table 1 provides descriptive statistics and correlations of variables averaged across all 24 trials. Age was correlated with performance on the matrix completion trials and was also positively related to the processing strategies of encoding, integrating, and completing the model (toggling). That is, older children solved more problems correctly, and they were more likely to encode and integrate the relations and to toggle more between the Matrix and Response Spaces. Furthermore, problem-solving performance was also correlated with the use of each processing strategy.

Age differences. Figure 2 illustrates younger (5- and 6-year-olds) and older children’s (7- and 8-year-olds’) matrix completion problem-solving trial by trial. To examine possible age differences and possible improvement with experience, children’s problem-solving performance on the 24 trials was divided into four phases (with 6 trials within each phase). A 2 (age: young vs. old) $\times$ 4 (phase) ANOVA was performed on the number of trials in which children correctly solved the matrix problems in each phase. The ANOVA reveals the main effect of age, $F(1, 67) = 22.27$, $p < .0001$, $MSE = 10.71$, $\eta^2 = .250$, but not phase, nor interaction, indicating that older children outperformed the younger group in solving the problems, and that neither the older nor the younger children improved their performance over phases.

Additional analyses were conducted to examine what types of errors participants made and how they gazed when searching for the correct answer within the Response Space. The mean number of verbal responses for Correct, Incorrect–One Relevant Rule (averaged over the two items), Incorrect–One Irrelevant Rule, and

Table 1
A Correlation Matrix Between Problem Solving Performance and Processing Strategies in Experiment 1

Variables	1	2	3	4	5	6	Mean	*SD*
1. Age (months)	—						82.98	13.81
2. Total viewing duration (ms)	−.01	—					144,741.55	44,834.19
3. Number of trials on which encoding occurred	.43**	.57**	—				6.30	5.15
4. Number of trials on which integrating occurred	.19	.65**	.68**	—			.46	.88
5. Mean toggles	.32**	.63**	.30*	.40**	—		4.54	1.49
6. Overall number of correct responses	.60**	.43**	.60**	.37**	.41**	—	14.42	7.50

* $p < .05$. ** $p < .01$.

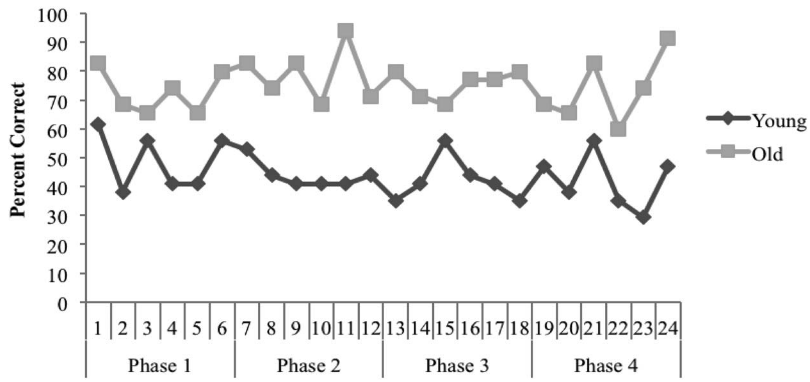

Figure 2. Five- and 6-year-olds' and 7- and 8-year-olds' matrix completion problem-solving performance over the four phases in Experiment 1.

Incorrect–Relevant and Irrelevant Rules (averaged over the two items) were 10.65, 5.01, 4.90, and 3.44, respectively, for younger children, and 18.03, 3.24, 1.76, and 0.97, respectively, for older children. A 2 (age) × 3 (type of error) ANOVA was performed on the number of trials in which children selected each incorrect option. The correct answer was not included in this analysis because it was analyzed above as a key problem-solving index, and because all types of options added up to 100%. The ANOVA revealed the main effect of age, $F(1, 67) = 21.48$, $p < .001$, $MSE = 14.58$, $\eta^2 = .243$, error types, $F(2, 134) = 36.85$, $p < .001$, $MSE = 1.75$, $\eta^2 = .355$, and interaction between age and error type, $F(2, 134) = 4.62$, $p < .05$, $MSE = 1.75$, $\eta^2 = .065$. These results suggest that children were more likely to select Incorrect-Relevant and Irrelevant Rules items than other types of incorrect items, and as compared to older children, younger children were more likely to do so.

The eye-tracking data within the Response Space revealed a pattern that is consistent with children's verbal reports of their choices. Average numbers of fixations on the four types of options within the Response Space were 3.02, 2.47, 2.31, and 1.07, respectively, for younger children, and 3.84, 2.99, 2.04, and 0.72, respectively, for older children. A 2 (age) × 4 (option types) ANOVA was performed on number of fixations per item. The ANOVA revealed a marginal effect of age, $F(1, 67) = 3.53$, $p = .065$, $MSE = 0.64$, $\eta^2 = .050$, the main effect of option types, $F(3, 201) = 152.58$, $p < .001$, $MSE = 0.52$, $\eta^2 = .695$, and an interaction between age and option type, $F(3, 201) = 11.29$, $p < .001$, $MSE = 0.52$, $\eta^2 = .144$.

These eye-tracking error patterns within the Response Space help verify in some way our speculation that the gaze shifts between the Matrix and Response Spaces reflect the completing the model process. Successful participants tended to gaze more often at the correct item, and they rarely went back to the distractors as compared to items that more closely resembled the correct item. This suggests that the completing the model process is a cognitive strategy that helped them eliminate those options that did not match their constructed mental model, and confirmed the correct item by checking back to the Matrix Space and its visual rules. Presumably solvers did keep the representations of the relations in mind and looked back and forth in order to check whether a particular item matched both of the rules when they searched for an appropriate item to complete the matrix.

Relations between problem-solving performance and processing strategies. We then further examined the relations between children's problem-solving performance and processing strategies. Using the problem-solving performance median (over a total of 24 trials) for each age group, two groups were formed at each age level: low and high performance groups. The means (and *SD*s) for younger-low performance, younger-high performance, older-low performance, and older-high performance groups were 4.24 (1.79), 17.06 (5.08), 14.06 (5.34), and 21.89 (0.96), respectively.

Figure 3 (Panel A) illustrates use of encoding by the four groups. To examine the prevalence of encoding in the four groups, a 2 (age: younger vs. older) × 2 (performance: low vs. high) ANOVA was performed on the number of trials during which encoding occurred. The ANOVA revealed main effects of age, $F(1, 65) = 11.89$, $p < .001$, $MSE = 19.74$, $\eta^2 = .155$, and performance, $F(1, 65) = 13.45$, $p < .0001$, $MSE = 17.62$, $\eta^2 = .171$, indicating that older children and those who solved the matrix problems more effectively were more likely to use encoding than younger children and those who did not do well in solving the problems, respectively. No significant interaction was revealed. Panel B depicts the integrating strategy used by the children in these four groups.

A separate 2 (age: younger vs. older) × 2 (performance: low vs. high) ANOVA was performed on integrating relations across the two dimensions. The ANOVA revealed the main effect of problem-solving performance, $F(1, 65) = 7.74$, $p < .01$, $MSE = 0.72$, $\eta^2 = .106$, but not age, nor the interaction. Figure 3 (Panel B) depicts the integrating strategy used by the children in these four groups.

To examine the use of the completing the model component, a 2 (age: younger vs. older) × 2 (performance: low vs. high) ANOVA was performed on the mean number of toggles that occurred on the trials. The ANOVA revealed the main effect of performance, $F(1, 65) = 8.07$, $p < .01$, $MSE = 1.83$, $\eta^2 = .110$, and an interaction between performance and age, $F(1, 65) = 7.53$, $p < .01$, $MSE = 1.83$, $\eta^2 = .104$. Post hoc analyses indicate that the performance effect on toggling was evident only at the younger

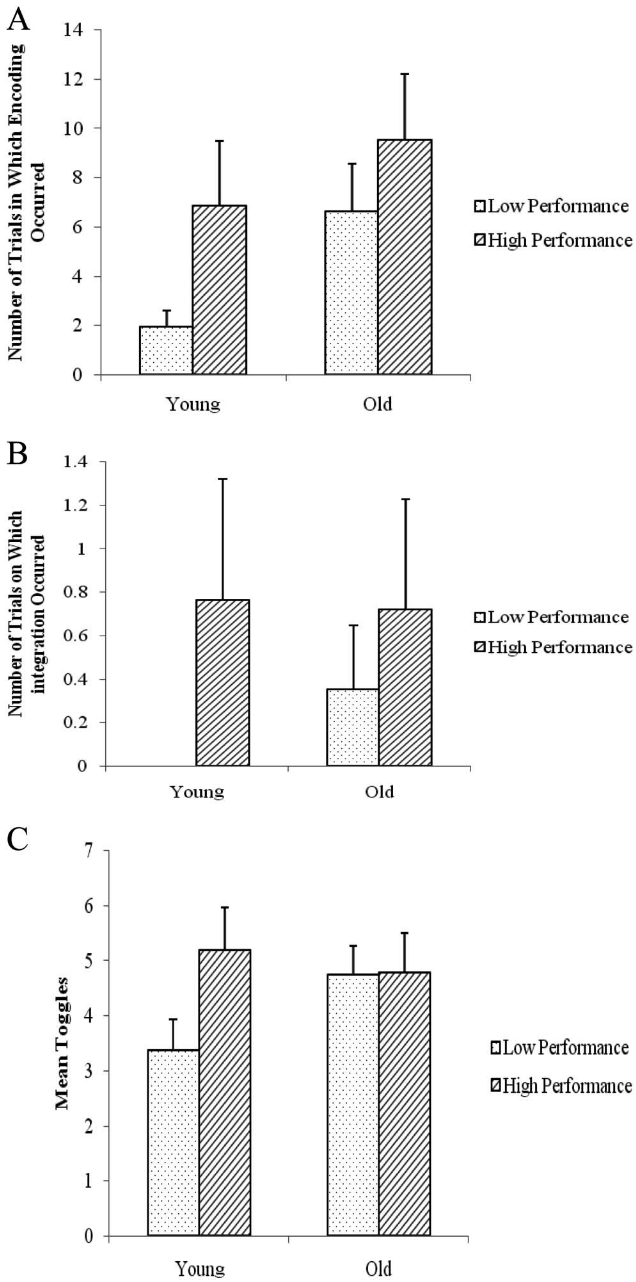

Figure 3. Children's strategy use in encoding (Panel A), integrating (Panel B), and toggling (Panel C) in Experiment 1.

level, $F(1, 66) = 15.11$, $p < .001$, $MSE = 1.87$, $\eta^2 = .186$. Figure 3 (Panel C) illustrates use of the toggling strategy by the children in these four groups.

Overall, this experiment demonstrated that 7- and 8-year-old children outperformed 5- and 6-year-olds in solving the matrix completion tasks and that the developmental differences in problem-solving performance were largely due to differences in engaging the processing strategies hypothesized to facilitate problem-solving performance. Encoding and integrating the dimensions of the relations and completing the model were evident as critical processing strategies, and their use greatly facilitated solving the matrix completion problems. Still, an issue remains unexamined: Would 5- and 6-years olds, whose poor performance has been consistently demonstrated in solving matrix tasks, be capable of learning to solve matrix completion problems if given adequate feedback? Experiment 2 was designed to explore the role of feedback in solving the tasks. Furthermore, we asked whether providing feedback would also improve young children's use of the relevant processing strategies.

Experiment 2

Method

Participants. Thirty-three 5- and 6-year-old children (ranging from 59 to 80 months, $M = 70$ months, $SD = 5.90$) participated in the experiment. Fourteen children (42%) were female. All parents reported normal vision and none of the children wore glasses. An additional four children participated but were dropped from the final sample due to their inability to engage or experimenter's error.

Materials and procedure. Materials were identical to those used in Experiment 1. All procedural details were the same as well, with the exception of the feedback provided following each trial. After the child completed each trial, the experimenter guided the child to attend to the relations by pointing out the rule underlying the relation in the vertical and horizontal directions: "See, this is getting darker, and this is getting smaller, so this one (pointing at the correct item) is the right choice."

Results

Data reduction and analysis. Data reduction and analyses were identical to Experiment 1. In this experiment, 20 trials of eye-tracking data from trials with less than 1,000 ms of total viewing time were eliminated. This reflected a 2.5% loss overall (out of 792 total trials). Measures for problem-solving performance and processing strategies were the same as those in Experiment 1.

Data analyses were performed to answer the following questions: (a) Would providing feedback during problem solving facilitate 5- and 6-year-olds' solving the matrix completion tasks as compared to children at the same age who received no feedback in Experiment 1? (b) Would feedback also enhance online processing strategies during problem solving? (c) Would children's processing strategies on the first phase be associated with their problem-solving performance on the later phases?

Effects of feedback on problem-solving performance. To examine the possible effects of feedback on problem-solving performance, we labeled the data of 5- and 6-years olds in Experiment 1 as the No Feedback condition, and we compared their performance to children of the same age in Experiment 2, who received explicit verbal feedback. Figure 4 illustrates children's problem solving trial by trial in each condition. A 2 (Feedback: yes vs. no) × 4 (Phase) ANOVA was performed on the number of trials in which children correctly solved the matrix problems on each phase. The ANOVA revealed no main effects, but the interaction was significant, $F(3, 195) = 7.67$, $p < .0001$, $MSE = 1.14$, $\eta^2 = .106$. A one-way ANOVA on each condition revealed that children

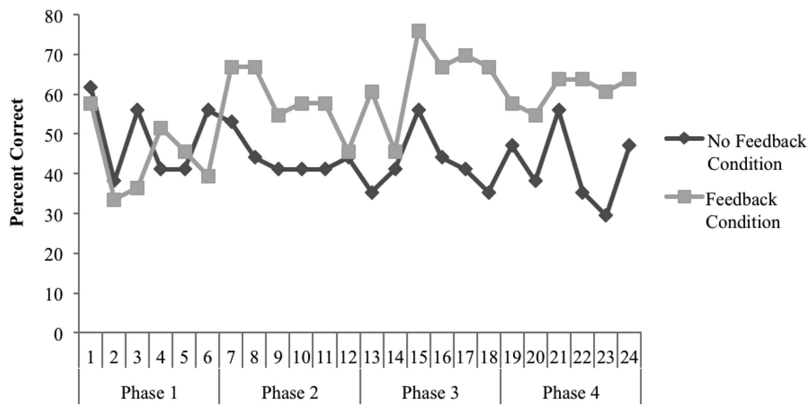

Figure 4. Five- and 6-year-olds' matrix completion problem-solving performance over the four phases in the No Feedback condition (in Experiment 1) and Feedback condition (Experiment 2).

in the Feedback condition improved their performance in solving the problems over the four phases, $F(3, 96) = 7.73$, $p < .001$, $MSE = 1.21$, $\eta^2 = .195$, whereas those in the No Feedback condition did not. These results demonstrate the positive effects of feedback on problem-solving performance, and indicate that with feedback, 5- and 6-year-olds were able to learn to solve the matrix completion tasks.

Error analyses were also conducted to examine the effect of feedback on error types. As indicated in Experiment 1 for younger children in the No Feedback condition, the mean number of verbal responses for Correct, Incorrect–One Relevant Rule, Incorrect–One Irrelevant Rule, and Incorrect–Relevant and Irrelevant Rules were 10.65, 5.01, 4.90, and 3.44, respectively, as compared to the Feedback condition (Experiment 2), 13.51, 4.88, 3.61, and 2.00, respectively. A 2 (condition) × 3 (type of error) ANOVA revealed the main effect of error type, $F(2, 130) = 40.02$, $p < .001$, $MSE = 2.17$, $\eta^2 = .381$, a marginal effect of condition, $F(1, 65) = 2.83$, $p = .098$, $MSE = 16.26$, $\eta^2 = .042$, and an interaction between condition and error type, $F(2, 130) = 3.93$, $p < .05$, $MSE = 2.17$, $\eta^2 = .057$.

Similarly, average numbers of fixations on these four types of options within the Response Space were 3.02, 2.47, 2.31, and 1.07, respectively, in the No Feedback condition, as compared to 3.59, 2.98, 2.35, and 0.99, respectively, in the Feedback condition. A 2 (condition) × 4 (option type) ANOVA revealed the main effect of condition, $F(1, 65) = 6.11$, $p < .05$, $MSE = 0.74$, $\eta^2 = .086$, and option type, $F(3, 201) = 114.04$, $p < .001$, $MSE = 0.55$, $\eta^2 = .637$, and an interaction between condition and option type, $F(3, 201) = 3.35$, $p < .05$, $MSE = 0.55$, $\eta^2 = .049$. The different patterns between conditions resembled those between younger and older children in Experiment 1, suggesting that feedback helped participants focus on items that were more similar to, and ignore items that were different from, the correct object.

Effects of feedback on processing strategies. We further examined whether providing feedback improved young children's use of the processing strategies. Children in each condition were divided into two groups based on their problem-solving performance over the last three phases (a total of 18 trials), given that children's initial performances were similar (on the first phase) in these two conditions, and children in the Feedback condition did not improve their performance before the second phase. The means (and *SD*s) for the Feedback condition, low performance and high performance groups, and the No Feedback condition, low performance and high performance group, are 6.59(3.32), 15.63(1.54), 2.88(1.73), and 12.53(4.23), respectively.

To examine the differences in encoding, a 2 (condition: feedback vs. no feedback) × 2 (performance: low vs. high) ANOVA was performed on encoding. The ANOVA revealed main effects of condition, $F(1, 63) = 5.361$, $p < .05$, $MSE = 8.36$, $\eta^2 = .078$, and performance, $F(1, 63) = 17.90$, $p < .001$, $MSE = 8.36$, $\eta^2 = .221$, indicating that children in the Feedback condition and those who solved the matrix problems more effectively were more likely to use the encoding strategy than those in the No Feedback condition and those who did not do well in solving the problems, respectively. No significant interaction was revealed. Figure 5 (Panel A) illustrates the encoding strategy used by the children in these four groups (combined over the last three phases).

To investigate the use of integration, a separate 2 (condition: Feedback vs. No Feedback) × 2 (performance: low vs. high) ANOVA was performed on integrating relations across the two dimensions. The ANOVA revealed the main effects of condition, $F(1, 63) = 6.99$, $p < .05$, $MSE = 0.53$, $\eta^2 = .100$, and problem-solving performance, $F(1, 63) = 6.99$, $p < .05$, $MSE = 0.53$, $\eta^2 = .100$, but not the interaction. Figure 5 (Panel B) depicts the integrating strategy used by the children in these four groups.

Finally, the use of the completing the model strategy was examined with a 2 (condition: Feedback vs. No Feedback) × 2 (performance: low vs. high) ANOVA performed on mean number of toggles. The ANOVA reveals the main effects of condition, $F(1, 63) = 18.65$, $p < .0001$, $MSE = 2.72$, $\eta^2 = .228$, and performance, $F(1, 63) = 17.90$, $p < .0001$, $MSE = 2.72$, $\eta^2 = .221$. The interaction between condition and performance was not significant. Children in the Feedback condition and those that performed better demonstrated more toggling than those in the No Feedback condition and those that performed less well. Figure 5 (Panel C) illustrates the toggling strategy used by the children in these four groups.

Initial use of processing strategies and subsequent problem-solving performance. To examine whether children's processing strategy use on the earlier phase (Phase 1) is associated with

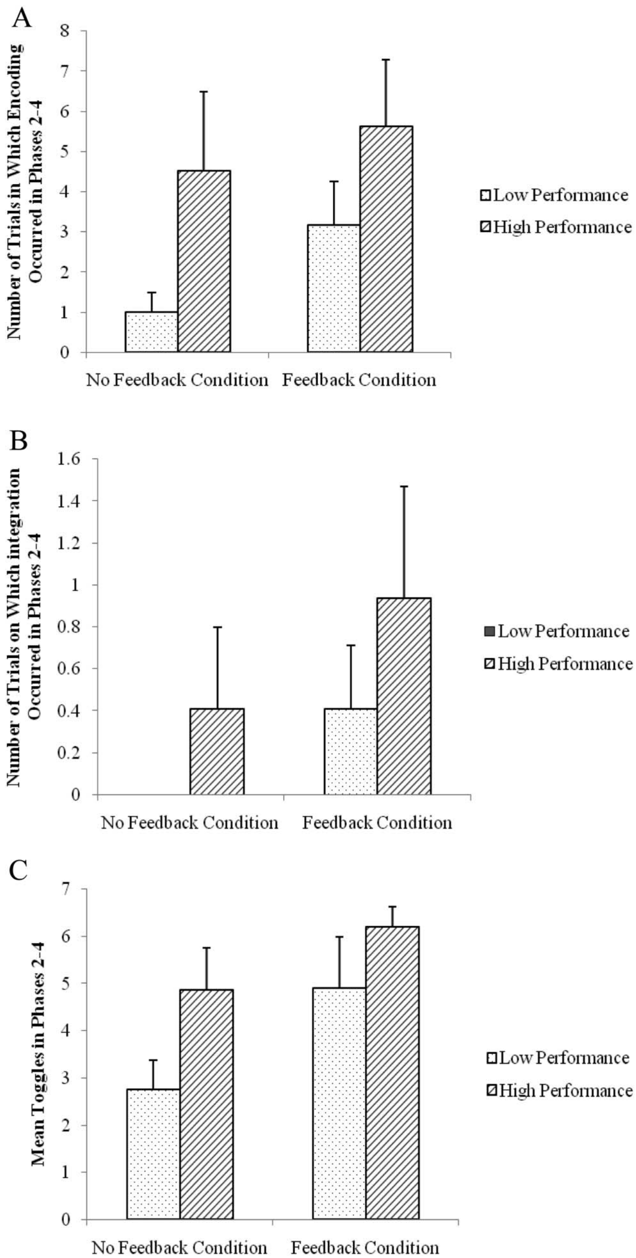

Figure 5. Children's strategy use in encoding (Panel A), integrating (Panel B), and toggling (Panel C).

their problem-solving performance on the later phases (2–4), we first categorized children's problem-solving performance over Phases 2 to 4 into three performance groups in each condition: Children in the Low Performance group were those who performed at the random level (with 4 or fewer correct out the 18 trials, or 20% correct), and children in the High Performance group were those who had 14 or more correct out the 18 trials (80% correct). The rest of the children were in the Intermediate Performance group. Table 2 shows the distribution of numbers of 5- and 6-year-old children in each condition in the High, Intermediate, and Low Performance groups. This pattern is consistent with the interpretation of the feedback effects on children's problem-solving performance, $\chi^2(2) = 4.78$, $p = .092$.

To determine the relations between the processing strategies on the first phase (before the children in the Feedback condition started to improve their problem-solving performance) and their later problem-solving performance, we examined whether and how children in these three groups (combined over the two conditions) used the processing strategies on Phase 1. Figure 6 shows that children in the High, Intermediate, and Low Performance groups differed in their initial use of the encoding (Panel A, $F(2, 64) = 9.19$, $p < .001$, $MSE = 1.56$, $\eta^2 = .223$), integrating (Panel B, not significant due to few trials in which integrating occurred) and toggling processes (Panel C, $F(2, 64) = 2.96$, $p = .059$, $MSE = 3.53$, $\eta^2 = .085$) on Phase 1.

A stepwise regression analysis was conducted on the combined data from Experiments 1 and 2 to determine the degree of independent contribution of each of the three hypothesized processes to matrix problem-solving performance. The predictors were number of trials on which encoding occurred, number of trials on which integration occurred, and mean toggles, as well as age (months). The dependent variable was the number of trials on which the matrix problem was successfully solved. Encoding, age, and toggling predicted matrix problem solving: these three variables accounted for 48% of the variance in performance. Encoding accounted for 32% of the variance in matrix problem solving, age accounted for an additional 11%, and toggling accounted for an additional 5%. It is not surprising that integration was not a significant predictor given the high correlation between encoding and integrating: integration is possible only when encoding is achieved.

Overall, data analyses in Experiment 2 (Feedback condition with 5- and 6-year-olds) and in Experiment 1 (No Feedback condition with 5- and 6-year-olds) showed the role of feedback received at the end of each trial in improving young children's problem-solving performance as well as their processing strategies over phases. These results further confirmed that encoding and integrating the relations and completing the model are critical strategies that underlie problem solving involved in relational reasoning. Providing feedback facilitated young children's use of processing strategies through which problem-solving performance was improved. Furthermore, the use of processing strategies also affected children's subsequent learning in solving the matrix completion tasks.

Table 2
Distribution of Numbers of 5- and 6-Year-Old Children in Each Condition in the High, Intermediate, and Low Performance Groups

	Performance group			
	Low	Medium	High	Total
Condition				
No Feedback (*N*)	13	12	9	34
Feedback (*N*)	5	14	14	33
Total (*N*)	18	26	23	67

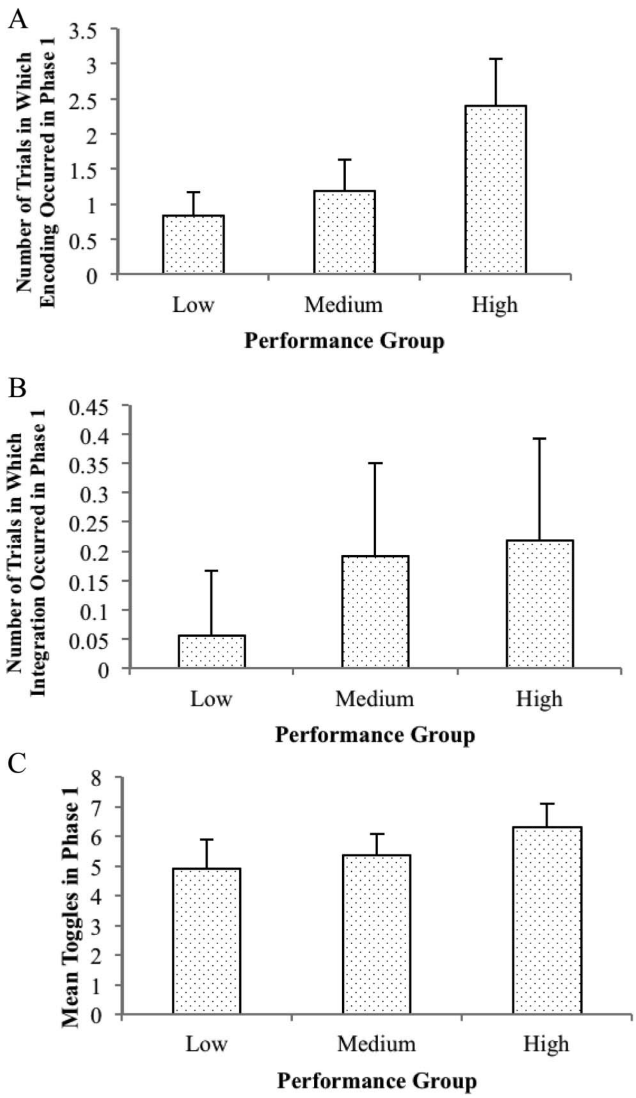

Figure 6. Children initial use of encoding (Panel A), integrating (Panel B), and Toggling (Panel C) on Phase 1 in the High, Medium, and Low performance groups on Phases 2–4.

General Discussion

This study was designed to examine the underlying processes and mechanisms of children's relational problem solving using an eye-tracking method, a microgenetic approach, and a componential analysis. The experiments yielded the following key findings: (a) Children between the ages of 5 and 6 (the younger group) and 7 and 8 (the older group) experienced a dramatic change in their ability to solve the matrix completion tasks as well as in their use of the processing strategies that underlie the problem solving. (b) With feedback, young children (5- and 6-year-olds) were able to learn to solve relational reasoning tasks over a short time frame. (c) Developmental differences and the effects of feedback on problem-solving performance were associated with the improved use of processing strategies, including encoding relations, integrating dimensions, and completing the model: the children who solved the problems effectively were those who used these strategies. Furthermore, children's initial use of processing strategies affected their subsequent learning; those children with more advanced processing strategies were more likely to solve the matrix problems. These findings and their theoretical and practical implications are discussed below.

Development and Learning in Relational Reasoning

Previous studies on various tasks and procedures reveal that children gradually acquire the ability to represent and map relations (e.g., Gentner & Rattermann, 1991; Kotovsky & Gentner, 1996; Richland, Morrison, & Holyoak, 2006). In the present study, 5-year-olds solved the matrix problems basically at the chance level. While only a small proportion of 6-year-olds showed proficiency in solving the problems, a majority of 7- and 8-year-olds solved the matrix completion tasks effectively. Error analyses also revealed that children paid more attention to those items that were more similar to the correct item in terms of the two relevant rules. This pattern of gazing within the Response Space was especially evident in older children's matrix problem solving. This developmental difference is consistent with findings reported in earlier studies (e.g., Overton & Brodzinsky, 1972; Parker & Day, 1971; Siegler & Svetina, 2002).

The present study also showed the effects of the feedback provided at the end of each trial on solving the matrix completion tasks, demonstrating that children's thinking becomes more relational with age and experience. Feedback that highlighted the relational information provided the scaffolding needed to guide young children toward understanding the componential processes required to solve the relational reasoning tasks. The role of feedback in 5- and 6-year-olds' problem-solving performance indicates that their difficulty in solving the matrix completion tasks was not due to any incapacity to represent the problem structure. Rather, young children failed to utilize the processing strategies crucial to completing the matrix. The primary purpose of the present study was to examine the cognitive processes and mechanisms that account for children's learning to solve matrix completion problems. The use of an eye-tracking approach allowed us to assess in a novel way children's online cognitive processing strategies during their problem solving. By analyzing their eye-gaze and -movement paths, we were able to gain insights into which processing components children engaged as they encountered the problems, how children's performance was influenced by these processing components, and how children's strategy changes influenced their problem-solving performance.

Processing Components Involved in Solving Relational Reasoning Tasks

The first two processing components necessary to solving relational reasoning problems involve encoding and integrating the target rules of the matrix. These components are important for several reasons. Accurate choice from the alternatives required the identification and use of the relational information. In order to solve the relational tasks successfully, children needed to encode effectively by knowing which information was critical to attend to and use. The problem solver needed to construct the model by integrating relations along multiple dimensions (i.e., construct the rules or relations between the objects in the rows and columns). We assumed relational encoding when eye-tracking measurements showed that a child had scanned the objects either horizontally or vertically, and we assumed relational integration when a child scanned the objects both horizontally and vertically within a single trial.

The impact of encoding and integrating on problem solving was evident in the present study. First, as a group, older children who solved the matrix problems were more likely to encode and integrate the relations. Second, analyses of individual differences in children's problem-solving performance indicated that those children who correctly solved the problems were more likely to use encoding and integrating strategies. Third, feedback that highlighted the matrix relations facilitated children's use of these processing strategies and thus improved their problem-solving performance, as compared to young children in the No Feedback condition. Fourth, those children who engaged in encoding and integrating strategies on the first phase were more likely to solve the subsequent problems of the second to fourth phases successfully. These results provide converging evidence that encoding relations and integrating rules are critical processing strategies for successfully solving relational reasoning tasks.

A final processing strategy necessary for solving relational reasoning tasks involves completing the model constructed by searching for a relationally matched item. In the matrix completion task, after integrating the rules, the child needed to search the Response Space for a match to the mental model he or she had constructed. While the model was being held in working memory, a solver needed to look back and forth between the Matrix Space and the Response Space in order to eliminate unmatched items and confirm the matching item. We examined how individual differences in the completing the model process impacted children's success in solving the matrix completion tasks. The results demonstrated the role of inspecting the Matrix and Response Spaces back and forth (i.e., children's gaze moved from the matrix area to the response area and vice versa). This eye-gaze strategy proved to be important for successful problem solving: (a) the completing the model process (as reflected in toggling) was an additional factor that contributed to matrix problem solving, as shown in the stepwise regression analysis; (b) children who successfully solved the matrix completion problems were more likely to use this eye-gaze strategy; (c) providing feedback that highlighted the matrix relations had significant effects upon children's use of the strategy as well as their problem-solving performance; and (d) younger children's problem-solving performance was especially associated with the completing the model process. The use of the completing the model process appeared to be more important for younger children's problem-solving performance, presumably due to young children's more limited working memory capacity. It was likely challenging for young children to hold the relational representation in working memory while searching the Response Space, making it more important to check back and forth between the Matrix and Response Spaces. While the first two processes, encoding and integrating, involve representing the relational rules that are necessary for solving the problems, the completing the model process involves a cognitive strategy of applying the constructed relational rules to search for a correct item to complete the model.

In sum, encoding relations, integrating rules, and completing the model were evident in the present study as processing components essential for solving the matrix tasks. Encoding relevant relations and integrating rules were initial steps toward accurate representations that could guide children's relational problem solving across various relational reasoning tasks. Children's ability to complete the model by searching for correct options proved to be another critical processing strategy that facilitated effective relational reasoning. These domain-independent processing strategies are likely to be critical for children in order to solve various relational reasoning problems, such as conservation tasks (Gelman, 1969), classic analogy problems (Goswami & Brown, 1990), transitive inference problems (Bryant & Trabasso, 1971), and spatial symbolic understanding tasks (Chen, 2007; DeLoache, 2000; Loewenstein & Gentner, 2001; Newcombe et al., 1998; Newcombe & Huttenlocher, 2000).

Learning and Generalizing Processing Strategies for Reasoning

A critical component of learning involves generalizing acquired processing strategies across tasks. The present experiments demonstrated that children as young as 5 and 6 years are capable of learning to solve the matrix completion tasks by generalizing the acquired strategies across trials/tasks. In the present study, the demonstrated role of feedback in improving young children's relational problem solving is consistent with previous findings, which indicated both that providing explicit verbal rules helped 6-year-olds succeed on matrix completion tasks, and that receiving the correct answer and being asked to explain why this answer was right also enabled 6-year-olds to solve the subsequent problems effectively (Siegler & Svetina, 2002).

When learning a new approach to solving the matrix reasoning tasks, 5- and 6-year-olds demonstrated the ability to generalize strategies across superficially different trials/tasks, which differed in specific objects and types of relations (e.g., shade, size, item, and orientation). Successful transfer of an acquired strategy from one task to another reflects the construction of a general approach, rather than a strategy embedded in the original items and their superficial features (Chen, 2007). Such a general approach enables children to map from one task to another. In the present Experiment 2's Feedback condition, 5- and 6-year-olds did not appear to increase their performance during the first phase; in other words, it took several trials for the feedback to take effect, suggesting that young children's initial representation of the task was relatively specific and their strategies tied to specific tasks with specific objects and relations. Only after experiencing multiple trials/tasks with feedback were young children able to achieve a more abstract scheme, presumably because they had the opportunity to compare multiple structurally similar instances or problems (e.g., Gentner & Namy, 1999). This comparison process can lead to the subsequent highlighting of common underlying relational structures shared across problems (Kurtz et al., 2001; Loewenstein, Thompson, & Gentner, 1999). Thus, the gradual change of strategies reflects the process of establishing a more abstract representation that enables its generalization across specific instances.

It is worth noting that while the present study was not specifically designed to examine optimal feedback, it nevertheless demonstrated that guiding the child to attend to the relations by highlighting the rule underlying the relation in the vertical and horizontal directions effectively facilitated the encoding and integrating processes involved in solving relational reasoning tasks. The source of strategy change is an important theoretical and practical issue involved in learning and development (e.g., Chen & Klahr, 1999; Chen & Siegler, 2000; Rittle-Johnson, 2006; Siegler, 2005, 2006; Siegler & Chen, 1998, 2008). It would be fruitful to explore further which feedback or training approaches are optimal for promoting children's relational reasoning, and what precise mechanisms are involved in facilitating children's processing strategies in solving relational reasoning tasks.

Conclusions

The current research examines how children at different ages solve matrix completion tasks, and explores how young children come to understand and use processing strategies in solving relational reasoning tasks. This study demonstrates the utility of the eye-tracking approach in exploring the underlying processes and mechanisms of children's problem solving and reasoning. This study also helps fill in some gaps in our understanding of the processes involved in solving relational reasoning problems. It demonstrates that developmental differences in children's problem-solving performance on such problems are largely due to deficiencies in engaging the processing strategies critical for sound reasoning. It also shows that instructional interventions in which young children were directed toward relational information of the matrix completion problems improved their processing strategies and problem-solving performance. It is evident that encoding and integrating rules, completing the model, and generalizing strategies across analogous tasks are critical processing strategies at the core of relational thinking and learning. The same component processes needed to solve the matrix completion problems seem likely to be critical for solving a wide variety of problems involving relational reasoning. Examining these processing strategies would appear to be essential for a comprehensive understanding of children's development and learning of relational thinking, a central ability in human higher-order cognition.

References

Alibali, M. W. (1999). How children change their minds: Strategy change can be gradual or abrupt. *Developmental Psychology, 35,* 127–145. http://dx.doi.org/10.1037/0012-1649.35.1.127

Bethell-Fox, C. E., Lohman, D. F., & Snow, R. (1984). Adaptive reasoning: Componential and eye movement analysis of geometric analogy performance. *Intelligence, 8,* 205–238. http://dx.doi.org/10.1016/0160-2896(84)90009-6

Bryant, P. E., & Trabasso, T. (1971). Transitive inferences and memory in young children. *Nature, 232,* 456–458.

Carlson, J. S., & Wiedl, K. H. (1977). Modes of information integration and piagetian measures of concrete operational thought. *Intelligence, 1,* 335–342. http://dx.doi.org/10.1016/0160-2896(77)90016-2

Carpenter, P. A., Just, M. A., & Shell, P. (1990). What one intelligence test measures: A theoretical account of the processing in the Raven Progressive Matrices Test. *Psychological Review, 97,* 404–431. http://dx.doi.org/10.1037/0033-295X.97.3.404

Chen, Z. (2007). Learning to map: Strategy discovery and strategy change in young children. *Developmental Psychology, 43,* 386–403. http://dx.doi.org/10.1037/0012-1649.43.2.386

Chen, Z., & Klahr, D. (1999). All other things being equal: Acquisition and transfer of the control of variables strategy. *Child Development, 70,* 1098–1120. http://dx.doi.org/10.1111/1467-8624.00081

Chen, Z., & Klahr, D. (2008). Remote transfer of scientific-reasoning and problem-solving strategies in children. *Advances in Child Development and Behavior, 36,* 419–470. http://dx.doi.org/10.1016/S0065-2407(08)00010-4

Chen, Z., & Siegler, R. (2000). Across the great divide: Bridging the gap between understanding the thought of toddlers and older children. *Monographs of the Society for Research in Child Development, 65* (2), i–vii, 1–96.

DeLoache, J. S. (1987). Rapid change in the symbolic functioning of very young children. *Science, 238,* pp. 1556–1557.

DeLoache, J. S. (2000). Dual representation and young children's use of scale models. *Child Development, 71,* 329–338. http://dx.doi.org/10.1111/1467-8624.00148

Dimitrovsky, L., & Almy, M. (1975). Early conservation as a predictor of later reading. *The Journal of Psychology: Interdisciplinary and Applied, 90,* 11–18. http://dx.doi.org/10.1080/00223980.1975.9923919

Gelman, R. (1969). Conservation acquisition: A problem of learning to attend to relevant attributes. *Journal of Experimental Child Psychology, 7,* 167–187. http://dx.doi.org/10.1016/0022-0965(69)90041-1

Gentner, D. (1983). Structure-mapping: A theoretical framework for analogy. *Cognitive Science, 7,* 155–170. http://dx.doi.org/10.1207/s15516709cog0702_3

Gentner, D. (1988). Metaphor as structure mapping: The relational shift. *Child Development, 59,* 47–59. http://dx.doi.org/10.2307/1130388

Gentner, D. (1989). The mechanisms of analogical learning. In S. Vosniadou & A. Ortony (Eds.), *Similarity and analogical reasoning* (pp. 199–241). New York, NY: Cambridge University Press. http://dx.doi.org/10.1017/CBO9780511529863.011

Gentner, D., & Markman, A. B. (1997). Structure mapping in analogy and similarity. *American Psychologist, 52,* 45–56. http://dx.doi.org/10.1037/0003-066X.52.1.45

Gentner, D., & Medina, J. (1998). Similarity and the development of rules. *Cognition, 65,* 263–297. http://dx.doi.org/10.1016/S0010-0277(98)00002-X

Gentner, D., & Namy, L. L. (1999). Comparison in the development of categories. *Cognitive Development, 14,* 487–513. http://dx.doi.org/10.1016/S0885-2014(99)00016-7

Gentner, D., & Rattermann, M. (1991). Language and the career of similarity. In S. A. Gelman & J. P. Byrnes (Eds.), *Perspectives on language and thought: Interrelations in development* (pp. 225–277). New York, NY: Cambridge University Press. http://dx.doi.org/10.1017/CBO9780511983689.008

Gordon, P. C., & Moser, S. (2007). Insight into analogies: Evidence from eye movements. *Visual Cognition, 15,* 20–35. http://dx.doi.org/10.1080/13506280600871891

Goswami, U., & Brown, A. L. (1990). Melting chocolate and melting snowmen: Analogical reasoning and causal relations. *Cognition, 35,* 69–95. http://dx.doi.org/10.1016/0010-0277(90)90037-K

Halford, G. S. (1993). *Children's understanding: The development of mental models*. Mahwah, NJ: Erlbaum.

Hamel, B. R., & van der Veer, M. A. (1972). Structure d'ensemble, multiple classification, multiple seriation and amount of irrelevant information. *British Journal of Educational Psychology, 42,* 319–325.

Honomichl, R. D., & Chen, Z. (2006). Learning to align relations: The effects of feedback and self-explanation. *Journal of Cognition and Development, 7,* 527–550. http://dx.doi.org/10.1207/s15327647jcd0704_5

Hummel, J. E., & Holyoak, K. J. (1997). Distributed representations of structure: A theory of analogical access and mapping. *Psychological Review, 104,* 427–466. http://dx.doi.org/10.1037/0033-295X.104.3.427

Inhelder, B., & Piaget, J. (1958). The oscillation of a pendulum and the operations of exclusion. In B. Inhelder & J. Piaget; A. Parsons & S. Milgram (Trans.), *The growth of logical thinking: From childhoon to adolescence* (pp. 67–79). New York, NY: Basic Books. http://dx.doi.org/10.1037/10034-004

Inhelder, B., & Piaget, J. (1964). *The early growth of logic in the child: Classification and seriation*. London, UK: Routledge and Kegan Paul.

Jensen, A. R. (1987). Psychometric gas a focus of concerted research effort. *Intelligence, 11,* 193–198. http://dx.doi.org/10.1016/0160-2896(87)90005-5

Klahr, D., & Chen, Z. (2011). Finding one's place in transfer space. *Child Development Perspectives, 5,* 196–204. http://dx.doi.org/10.1111/j.1750-8606.2011.00171.x

Kotovsky, L., & Gentner, D. (1996). Comparison and categorization in the development of relational similarity. *Child Development, 67,* 2797–2822. http://dx.doi.org/10.2307/1131753

Kuhn, D. (1995). Microgenetic study of change: What has it told us? *Psychological Science, 6,* 133–139. http://dx.doi.org/10.1111/j.1467-9280.1995.tb00322.x

Kurtz, K. J., Miao, C. H., & Gentner, D. (2001). Learning by analogical bootstrapping. *Journal of the Learning Sciences, 10,* 417–446. http://dx.doi.org/10.1207/S15327809JLS1004new_2

Loewenstein, J., & Gentner, D. (2001). Spatial mapping in preschoolers: Close comparisons facilitate far mappings. *Journal of Cognition and Development, 2,* 189–219. http://dx.doi.org/10.1207/S15327647JCD0202_4

Loewenstein, J., Thompson, L., & Gentner, D. (1999). Analogical encoding facilitates knowledge transfer in negotiation. *Psychonomic Bulletin & Review, 6,* 586–597. http://dx.doi.org/10.3758/BF03212967

Markman, A. B., & Gentner, D. (1993). Structural alignment during similarity comparisons. *Cognitive Psychology, 25,* 431–467. http://dx.doi.org/10.1006/cogp.1993.1011

Marzolf, D. P., & DeLoache, J. S. (1994). Transfer in young children's understanding of spatial representations. *Child Development, 65,* 1–15. http://dx.doi.org/10.2307/1131361

Namy, L. L., Smith, L. B., & Gershkoff-Stowe, L. (1997). Young children's discovery of spatial classification. *Cognitive Development, 12,* 163–184. http://dx.doi.org/10.1016/S0885-2014(97)90011-3

Newcombe, N., & Huttenlocher, J. (2000). *Making space: Taking cognitive development one domain at a time*. Cambridge, MA: MIT Press.

Newcombe, N. S., & Huttenlocher, J. (2006). Development of spatial cognition. In D. Kuhn & R. S. Siegler (Eds.), *Handbook of child psychology* (6th ed., pp. 734–776). Hoboken, NJ: Wiley and Sons.

Newcombe, N., Huttenlocher, J., Drummey, A. B., & Wiley, J. G. (1998). The development of spatial location coding: Place learning and dead reckoning in the second and third years. *Cognitive Development, 13,* 185–200. http://dx.doi.org/10.1016/S0885-2014(98)90038-7

Newell, A., & Simon, H. A. (1972). *Human problem solving*. Englewood Cliffs, NJ: Prentice Hall.

Overton, W. F., & Brodzinsky, D. (1972). Perceptual and logical factors in the development of multiplicative classification. *Developmental Psychology, 6,* 104–109. http://dx.doi.org/10.1037/h0032204

Parker, R. K., & Day, M. C. (1971). The use of perceptual, functional, and abstract attributes in multiple classification. *Developmental Psychology, 5,* 312–319. http://dx.doi.org/10.1037/h0031424

Plumert, J. M., Ewert, K., & Spear, S. J. (1995). The early development of children's communication about nested spatial relations. *Child Development, 66,* 959–969. http://dx.doi.org/10.2307/1131791

Raven, J., Raven, J. C., & Court, J. H. (1998). Manual for Raven's Progressive Matrices and Vocabulary Scales. Section 4: The Advanced Progressive Matrices. San Antonio, TX: Harcourt Assessment.

Richland, L. E., Morrison, R. G., & Holyoak, K. J. (2006). Children's development of analogical reasoning: Insights from scene analogy problems. *Journal of Experimental Child Psychology, 94,* 249–273. http://dx.doi.org/10.1016/j.jecp.2006.02.002

Rittle-Johnson, B. (2006). Promoting transfer: Effects of self-explanation and direct instruction. *Child Development, 77,* 1–15. http://dx.doi.org/10.1111/j.1467-8624.2006.00852.x

Siegler, R. S. (1976). Three aspects of cognitive development. *Cognitive Psychology, 8,* 481–520.

Siegler, R. S. (2005). Children's learning. *American Psychologist, 60,* 769–778. http://dx.doi.org/10.1037/0003-066X.60.8.769

Siegler, R. S. (2006). Microgenetic analyses of learning. In W. Damon & R. M. Lerner (Series Eds.) & D. Kuhn & R. S. Siegler (Vol. Eds.), *Handbook of child psychology: Vol. 2: Cognition, perception, and language* (6th ed., pp. 464–510). Hoboken, NJ: Wiley.

Siegler, R. S., & Chen, Z. (1998). Developmental differences in rule learning: A microgenetic analysis. *Cognitive Psychology, 36,* 273–310. http://dx.doi.org/10.1006/cogp.1998.0686

Siegler, R. S., & Chen, Z. (2008). Differentiation and integration: Guiding principles for analyzing cognitive change. *Developmental Science, 11,* 433–448. http://dx.doi.org/10.1111/j.1467-7687.2008.00689.x

Siegler, R. S., & Crowley, K. (1991). The microgenetic method. A direct means for studying cognitive development. *American Psychologist, 46,* 606–620. http://dx.doi.org/10.1037/0003-066X.46.6.606

Siegler, R. S., & Jenkins, E. (1989). *How children discover new strategies*. Mahwah, NJ: Erlbaum.

Siegler, R. S., & Svetina, M. (2002). A microgenetic/cross-sectional study of matrix completion: Comparing short-term and long-term change. *Child Development, 73,* 793–809. http://dx.doi.org/10.1111/1467-8624.00439

Snow, R. E., Kyllonen, P. C., & Marshalek, B. (1984). The topography of ability and learning correlations. In R. J. Sternberg (Ed.), *Advances in the psychology of human intelligence* (Vol. 2, pp. 47–103). Mahwah, NJ: Erlbaum.

Sternberg, R. J. (1977). Component processes in analogical reasoning. *Psychological Review, 84,* 353–378. http://dx.doi.org/10.1037/0033-295X.84.4.353

Uttal, D. H. (2000). Seeing the big picture: Map use and the development of spatial cognition. *Developmental Science, 3,* 247–264. http://dx.doi.org/10.1111/1467-7687.00119

Received July 18, 2014
Revision received January 8, 2016
Accepted February 18, 2016 ■